MODELING DAILY CLOSING PRICE VOLATILITY USING SYMMETRIC GARCH

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Abstract

Modeling and forecasting the volatility of daliy closing price series is a significant area of financial econometrics since last few decades. Due to regional integration of the financial markets, investors not only interested in investing in their own countries stock markets but also investing in another countries stock markets. The aim of this study is to investigate the more volatile market and modelling the volatility.We use daily closing index of KSE-100 (Pakistan), BSESN (India) and CSE (Sri Lanka) as they are the member of SAARC countries covering the period 1st January, 2011 to 30th November, 2016. Empirical analysis shows that GARCH-in- mean model is found insignificant for BSESN and CSE. It reveals that there is no relationship between risk and expected return. Furthermore, CSE is more persistent stock market than the other two, but KSE-100 is highly volatile during the study period. GARCH-in-mean model with log variance in mean return equation is suggested for out-sample forecast of KSE-100. On the other hand, in CSE IGARCH and for BSESN any one from IGARCH and GARCH are suggested suitable model.

Keywords: Modeling , Stock Market, Empirical Analysis, Modeling Volatility.

JEL Classification: G100

Introduction

Modeling and forecasting financial data, such as stock market data, inflation rates, foreign exchange rates, etc. are very difficult task due to volatility. The nature and behaviour of the stock market returns are attractive to the researchers and market practitioner to forecast such variables. It has been observed that financial variables vary considerably, for some periods of time, though the forecast errors are relatively small but for other periods of time they are large. This suggested that the variation in forecast errors is not constant throughout, but varies from one period to another period, that is, there is some kind of autocorrelation present in the variance of forecast errors.

Over the past few decades for forecasting of financial time series and econometric, Box–Jenkins type of model (Autoregressive Integrated Moving Average (ARIMA)) was used in which

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variance is assumed to be constant. Modeling volatility is an imperative issue in stock markets and it has drawn the attention of researchers and market experts from the last few decades. There are numerous studies and different methods which have discussed the instability in financial series. As the financial data have non-concstant variance, Engle (1982) proposed the ARCH class of model to modeling conditional variance. Subsequently, Bollerslev (1986) improved ARCH model and developed Generalized ARCH (GARCH) model which avoid large lag length in conditional variance and gave more parsimonious model. Engle, Lilien and Robins (1987) and Glosten, Jagannathan, and Runkle (1993), explored that volatility cannot be directly observed, it has some characteristics such as volatility clustering and leverage effect etc.

Stock market is a financial institution of a country which provides an opportunity to individual and institutional investor for investment. Stock market plays an important contribution in the economic growth and development of a country. It is considered to be a primary barometer of a country's economic stability, therefore, rising prices related with the increased business asset. Stock markets promote the exchange of security between customer and dealer by minimizing the risk of investment. Basically, stock market consist of stocks and shares are issued by the company to its buyer to be one of the owners of the company. The overall market movement can be measured by the statistical composite measure called index.

With this perspective, this study compares the persistency of KSE-100 with BSESN and CSE and develops suitable conditional volatility forecast model for each stock market. In addition to this, it also investigated the relationship between the risk premimum parameter on its own volatility. For empirical estimation, data from 1st January, 2011 to 30th November 2016 a total of 1543 points of three South Asian countries, namely, India, Pakistan, and Sri Lanka are selected.

The outline of this paper is as follows. Section 2 provides a review of selected literature. Section 3 describes the brief introduction to the family of GARCH model, section 4 deals with data analysis and section 5 summarizes the conclusion.

Literature Review

P. Srinivasan (2011), applied FGARCH models namely GARCH, EAGRCH and TGARCH to forecast the conditional volatility of S& P500. According to their findings EGARCH and TGARCH model performed better than simple GARCH model.

The daily data of nineteen Arab countries were selected from 1st January 2000 to 19th November 2011 for modelling the exchange rate (Zakaria & Abdalla, 2012). Empirical analysis showed that 10 out of 19 countries currencies' representing volatility are an explosive process and also majority of currencies supporting that the negative shock follows high volatility for the next period than a positive shock.

Iulian and Ecaterina (2012), compared the changes in variance structure using seven Romanian trading companies data listed on the Bucharest Stock Exchange and indices of three markets, namely, BET, BET-XT and BET-C for the period 1997-2012, daily, weekly and monthly basis. GARCH-in–Mean model was applied to find the structural changes in volatility for selected periods with different frequencies and found that GARCH-in–Mean model performed well in weekly and monthly data. Şebnem and Fidan (2013) modelled daily returns of the Istanbul stock market using non-parametric GARCH model (Bühlmann & McNeill, 2002) instead of using parametric lagged values and their lagged volatility. The parameters were estimated using non-linear maximum likelihood method. They found that if the stock returns distribution is unknown or heavy tail then non-parametric GARCH model gave better estimates of the volatility. In the same year, Mohd. Aminul Islam (2013), used Family of Symmetric GARCH models for Asian markets such as KLSE (Malysia), JKSE (Indonesia) and STI (Singapore) to estimate the volatility. Furthermore, risk return relationship was also model via GARCH in mean process. Experimental analysis showd that all markets have positive risk return relationship. Moreover, Indonesian market was highly volatile than the other two selected markets.

The family of univariate GARCH; simple GARCH, Power-GARCH and component GARCH and multivariate GARCH-BEKK methods were applied to the polish economy (Fiszeder and Orzeszko, 2012). Forecasting performance of top markets of Asia, America and the United Kingdom were compared using symmetric and asymmetric models by Jiang and Forsberg (2012). Based on empirical analysis, it was found that selected models did not perform well due to economical, political and financial global changes.

Beside these, there are abundant literature which discussed risk-return relationship using GARCH-in-mean model, NYSE stock market by Bae et al. (2007) and Appiah and Menyah (2003) for eleven African stocks returns. A Hybrid financial system of KSE-100 index developed by Fatima and Hussain (2008), in their proposed system first they used the GARCH model to capture volatility and then the estimates of volatile model was given as input to ANN model. Their suggested hybrid system outplayed then the standard GARCH model and ANN model.

Introduction To Family Of GARCH Models

There are two types of Volatile models:

1. In the first category of volatile models, conditional variance changes over times as past errors leaving unconditional variance constant. ARCH and GARCH models are the example of this class.
2. The second category of volatile model is not purely the function of observation. These are called latent volatile or stochastic volatile models (Engle and Patton (2001)). These category of models can be used to explain structural breaks over random times and other factors such

as random amplitudes, multiple factors, jumps fat-tailed shocks, fractals and multi-fractals etc. They are also difficult to estimate and forecast.

GARCH

The Auto Regressive Conditional Hetroskedicity (ARCH) model introduced for modelling inflationary uncertainty, but has subsequently found especially wide use in the analysis of financial time series forecasting by Engle (1982). Coulson and Robins (1985) studied volatility of inflation and volatility in stock markets returns, Engle (1982) and Domowitz and Hakkio (1985) suggested that in models of inflation, large and small forecast errors appeared to occur in cluster.

ARCH (n) model is defined as,

$$\mathrm{k_t} = \phi_0 + \sum_{\mathrm{i=1}}^{\mathrm{n}} \phi_{\mathrm{i}} \alpha_{\mathrm{t-i}}^2$$

$$\mathrm{k_t} = \varphi_0 + (\varphi_1 G + \varphi_2 G^2 + \ldots + \varphi_n G^n)\ \alpha_t^2$$

$$\text{or} \quad k_t = \varphi_0 + \varphi(G)\ \alpha_t^2 \quad \ldots\ldots (1)$$

Where 'n' is the order of the ARCH process, is the innovation process with E [α_t] = 0 and autocorrelation $\mathrm{cov}(\alpha_t, \alpha_s) = 0$, $s \neq t$.

Equation '1' is called ARCH model and 'G' denotes the lag operators.

Empirically, ARCH model does not allow more lags in the conditional variance equation and typically imposed positivity constrain. Bollerslev (1986) invented GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model for representing long-memory and malleable lag structure both.

$$k_t^2 = \sum_{i=1}^{n} \varphi_i \alpha_{t-i}^2 + \sum_{j}^{m} \delta_j k_{t-j}^2 \quad \ldots\ldots (2)$$

$$k_t^2 = \varphi_0 + \varphi(G)\alpha_t^2 + \delta(G)k_t^2$$

Where

$m \geq 0$, $n > 0$,

$\varphi_0 > 0$, $\varphi_i \geq 0$

$i = 1, \ldots, n$

$\delta_j \geq 0$, $j = 1, \ldots, m$

Where equation (2) is called GARCH (m,n) process. For $m = 0$ equation (1) reduces to the ARCH (n) process.

GARCH–in–Mean model

In finance GARCH-in-mean (GARCH-M) model is improved form of the GARCH model as this model is well suited to account for the risk-return relationship, where the increased returns are expected with increased risk. It is observed that an increase in risk tends to conclude higher expected returns in share prices. To model such phenomenon GARCH-M model was introduced by Engle, Lilien and Robins (1987) and extended by Engle and Patton (2001). In GARCH-M conditional variance, standard deviation or logarithmic variance term is included into the mean equation.

If r_t is daily returns then the mean equation is define as,

$$r_t = \lambda_0 + \theta\left(\sigma_t^2, \sigma_t \text{ or } \log \sigma_t^2\right) + \alpha_t \quad \text{......................................} (3)$$

GARCH–M (m, n) can be defined as,

$$k_t^2 = \varphi_0 + \sum_{i=1}^{n} \varphi_i z_{t-1}^2 + \sum_{j=1}^{m} \delta_j k_{t-1}^2 \quad \text{......................................} (4)$$

Where λ_o and θ are constant. The constant θ is the risk premium parameter and the positive value shows that the return r_t has a positive relation to its own volatility. Equation (3) which is the conditional mean equation of the return shows an increase or decrease in the conditional variance is associated with mean return equation. GARCH–M model characterizes evolution of the mean and variance simultaneously in mean equation.

Integrated GARCH (IGARCH)

GARCH models assume that volatilities depend on past volatilities and also on past innovations. GARCH models are symmetric and have short memory. The Integrated GARCH (IGARCH) model modifies the GARCH model which incorporate an approximate unit root in the variance equation; i.e. $\sum_{i=1}^{n} \varphi_i + \sum_{j=1}^{m} \delta_j = 1$ (see Glosten et al. (1993)).

IGARCH (n, m) models design to modeled persistent changes in variance. Persistency is an important property of the volatile models which investigated how long shocks to conditional variance persist. Thus IGARCH model accounted maximum persistency as compared to GARCH.

Data Analysis And Results

In this study three stock markets daily closing price data are selected among the member of SAARC countries namely; KSE 100 of Pakistan, BSESN of India and CSE of Sri Lanka from Yahoo finance. We use data from 1st January, 2011 to 30th November, 2016 excluding weekends and then calculate returns of these closing price indices using the logarithmic transformation. The graphs of returns (Figure-1(b), 2(b) &3(b)) of closing indices of all markets show that continuously compound returns moving in both (positive and negative) direction around the mean and close to zero. Larger spike shows the leverage effects which separated very small fluctuation (volatility clustering).

Data from 1st January, 2011 to 23rd November, 2016 used for model building and 24th November, 2016 to 30th November, 2016 sample kept as a holdback period in order to compare out sample good or bad forecasting performance.

Figure1(a) Displays daily share data of BSESN

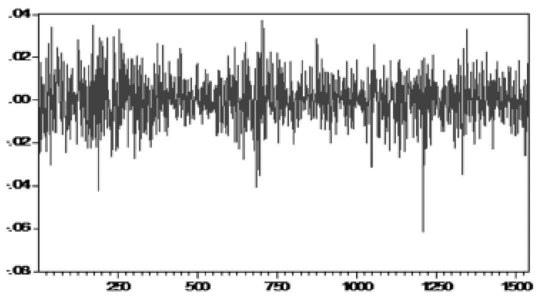

Figure 1 (b) Transformed data of BSESN

Figure 2 (a) Displays daily share data of KSE-100

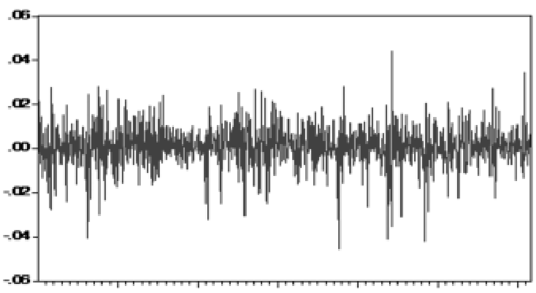

Figure 2 (b) Transformed data of KSE-100

Figure 3 (a) Displays daily share data of CSE

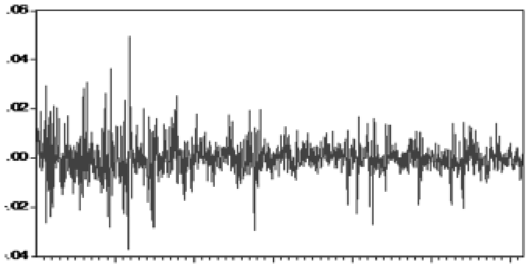

Figure 3 (b) Transformed data of CSE

Table 1
Provides general statistics of closing price returns of three stock markets

	BSESN	CSE	KSE-100
Mean	0.000168	-4.19E-05	0.00083
Median	0	0	0.00043
Maximum	0.037035	0.049567	0.044186
Minimum	-0.061197	-0.037227	-0.04558
Std. Dev.	0.010078	0.006818	0.008595
Skewness	-0.144424	0.122244	-0.393025
Kurtosis	4.807363	8.486484	6.270286
Jarque-Bera	215.2372	1937.863	726.8374
Probability	0	0	0

From Table 1, it can be seen that all three returns have positive mean value except CSE. Furthermore, mean return of KSE-100 is greater than BSESN and CSE. KSE-100 and BSESN are negatively skewed while CSE is positively skewed indicating the return distributions are asymmetric. Most importantly, all three markets have fat tail distribution suggesting excess kurtosis but CSE has much fatter tail than other two. Standard deviation is high in Bombay stock but minimum in Colombo stock. While experimenting, the Jarque–Bera test also rejected the null hypothesis of normality assumption.

In this section, we show model building process of GARCH, IGARCH and GARCH-M models. In order to select suitable model we used the Akaike Information Criterion (AIC) and Schwarz Bayesian Information Criterion (SBIC). The parameter estimated for the family of GARCH models using the maximum likelihood method under the hypothesis that errors are normally distributed. The empirical analysis shows following conclusions.

1. GARCH (1, 1) is found the suitable model in simple GARCH for all the series. From Table 2 sum of the coefficients of lagged squared residuals (ϕ_1) and lagged conditional variance (δ_1) for BSESN (0.98277), KSE-100 (0.86693) and CSE (0.9749) are highly statistically significant at 5% level of significance. Which shows that the conditional variance is highly persistent in all markets. Furthermore, BSESN as compared to other two selected stock markets returns is more persistent. The sum of $\phi_1 + \delta_1 < 1$ also explains that conditional volatilities are mean reverting process.

Table 2
Output of GARCH (1,1) model

Variance Equation				
$k_t^2 = \phi_0 + \phi_1 \alpha_{t-1}^2 + \delta_1 k_{t-1}^2$				
	ϕ_0	ϕ_1	δ_1	***p-v***
KSE-100	1.09E-05	0.2052	0.6617	0
S.E	(1.51E-06)	(0.222)	(0.0331)	
BSESN	1.69E-06	0.03826	0.94451	0
S.E	(5.93E-07)	(0.0085)	(0.0127)	
CSE	1.37E-06	0.1362	0.8387	0
S.E	(2.36E-07)	(0.01312)	(0.0136)	

Note: p-v indicates p-value and S.E represents standard error

2. In GARCH-M we used conditional standard deviation, variance and log variance in mean equation. Table (3, 4, & 5) represent risk premium θ parameter in mean equations is found statistically insignificant in CSE and BSESN, invalidating the assumption that there is correlation between risk and expected return. However, in KSE-100 θ is found significant at 5% level of significance shows that the return is positively related to its past volatility.

Table 3
Output of GARCH-M (1,1) with conditional standard deviation in mean equation

	Mean equation		Variance Equation			
	$r_t = \lambda_0 + \theta(\sigma_t) + \varepsilon_t$		$\mathrm{k}_t^2 = \phi_0 + \phi_1 \alpha_{t-1}^2 + \delta_1 \mathrm{k}_{t-1}^2$			
	λ_0	θ	φ_0	φ_1	δ_1	***p-v***
KSE-100	-0.000910	0.28526	1.16E-05	0.2110	0.6466	0
	S.E (0.0010) p-v [0.3719]	S.E (0.12992) p-v [0.0281]	S.E (1.62E-06)	S.E (0.0226)	S.E (0.03486)	
BSESN	-0.001478	0.1936	1.88E-06	0.0403	0.94056	0
	S.E (0.1533) p-v [0.3119]	S.E (0.1532) p-v [0.2066]	S.E (6.37E-07)	S.E (0.008972)	S.E (0.01345)	
CSE	2.87E-05	0.024358	1.37E-06	0.1363	0.8387	0
	S.E (0.00045) p-v [0.7759]	S.E (0.0856) p-v [0.9496]	S.E (2.38E-07)	S.E (0.0131)	S.E (0.0136)	

Note: p-v indicates p-value and S.E represents standard error.

Table 4
Output of GARCH-M (1,1) with conditional variance in mean equation

	Mean equation		Variance Equation			
	$r_t = \lambda_0 + \theta(\sigma_t^2) + \varepsilon_t$		$\mathrm{k}_t^2 = \phi_0 + \phi_1 \alpha_{t-1}^2 + \delta_1 \mathrm{k}_{t-1}^2$			
	λ_0	θ	φ_0	φ_1	δ_1	***p-v***
KSE-100	0.000371	14.75496	1.16E-05	0.211	0.6466	0
	S.E (0.000476) p-v [0.4365]	S.E (6.9759) p-v [0.0344]	S.E (1.62E-06)	S.E (0.0222)	S.E (0.0348)	
BSESN	-0.001478	0.1936	1.88E-06	0.040343	0.940567	0
	S.E (0.153292) p-v [0.3119]	S.E (0.1532) p-v [0.2066]	S.E (6.37E-07)	S.E (0.008972)	S.E (0.01345)	
CSE	6.44E-05	2.9401	1.37E-06	0.136255	0.838752	0
	S.E (0.000214) p-v [0.7637]	S.E (5.8032) p-v [0.6124]	S.E (2.38E-07)	S.E (0.0131)	S.E (0.0136)	

Note: p-v indicates p-value and S.E represents standard error.

Table 5
Output of GARCH-M (1, 1) with conditional log variance in mean equation

	Mean Equation		Variance Equation			
	$r_t = \lambda_0 + \theta \log(\sigma_t^2) + \varepsilon_t$		$\mathrm{k}_t^2 = \phi_0 + \phi_1 \alpha_{t-1}^2 + \delta_1 \mathrm{k}_{t-1}^2$			
	λ_0	θ	φ_0	φ_1	δ_1	***p-v***
KSE-100	0.01361	0.00126	1.17E-05	0.211633	0.6439	0
	S.E (0.00539) p-v [0.0116]	S.E (0.0005) p-v [0.0221]	S.E (1.63E-06)	S.E (0.022823)	S.E (0.03511)	
BSESN	0.008944	0.00092	1.87E-06	0.040343	0.940743	0
	S.E (0.007195) p-v [0.2317]	S.E (0.0072) p-v [0.2138]	S.E (6.33E-07)	S.E (0.0089)	S.E (0.0134)	
CSE	0.000546	3.66E-06	1.37E-06	0.1363	0.8387	0
	S.E (0.0028) p-v [0.7637]	S.E (2.38E-07) p-v [0.6124]	S.E (2.37E-07)	S.E (0.013128)	S.E (0.01356)	

Note: p-v indicates p-value and S.E represents standard error. In the IGARCH (1,1) the sum of ϕ_1 and δ_1 are equal to one. The sum of the parameter $\phi_1 + \delta_1$ is one in BSESN- and CSE reveals the more persistent as compared to KSE-100.

Table 6
Output of IGARCH

	Variance Equation		
	$\mathrm{k}_t^2 = \phi_1 \alpha_{t-1}^2 + \delta_1 \mathrm{k}_{t-1}^2$		
	φ_1	δ_1	***p-v***
KSE-100	0.0530	0.9469	0
	S.E (0.00244)	S.E (0.00244)	
BSESN	0.03222	0.9678	
	S.E (0.003747)	S.E (0.00375)	0
CSE	0.07894	0.9211	
	S.E (0.0038)	S.E (0.0038)	0

Note: p-v indicates p-value and S.E represents standard error.

Table 7 reports the in-sample FMSE of GARCH, IGARCH and GARCH-M models of three stock markets. GARCH- in- mean with conditional variance in mean equation has minimum root mean square error (RMSE) and mean absolute error (MAE). However, in BSESN and CSE have approximately same RMSE and MAE.

Table 7
In-sample forecast RMSE and MAE

	Model	(RMSE)	MAE
KSE-100	GARCH-M-Standard Deviation	215.68	143.748
	GARCH-M-variance	215.291	143.563
	GARCH-M-log variance	215.96	143.86
	IGARCH	215.962	143.756
	GARCH	216.287	144.359
BSESN	GARCH	217.544	159.484
	IGARCH	217.540	159.475
CSE	GARCH	43.131	28.447
	IGARCH	43.11786	28.4003

Table 8
Out-sample forecast of all stock markets

COUNTRY	Model	FRMSE
Karachi Stock Exchange (KSE-100)	GARCH(1,1)	175.4981
	GARCH-M(1,1)- StandardDeviation	164.093
	GARCH-M(1,1)-Variance	165.993
	GARCH-M(1,1)-Log Variance	162.372
	IGARCH(1,1)	169.2279
Bombay Stock Exchange (BSESN)	GARCH (1,1)	246.893
	IGARCH (1,1)	246.893
Colombo Stock Exchange (CSE)	GARCH (1,1)	9.2982
	IGARCH (1,1)	9.2398

From the above table GARCH-M-Log variance for KSE-100 and for CSE IGARCH have minimum FMSE. On the other hand, in BSESN GARCH and IGARCH both have same FMSE.

Conclusion

This study finds stable markets and also compares forecasting performance of daily closing prices indices of three stock markets KSE-100 of Pakistan, (BSESN) of India and Colombo Stock Exchange (CSE) of Sri Lanka using GARCH, GARCH-M and IGARCH models. We built above mentioned symmetric GARCH models for each stock markets by taking different values of parameter ('m' and 'n'), suitable models were selected based on minimum AIC and SBIC criterion. GARCH (1,1), IGARCH(1,1) and GARCH-M (1,1) are suitable models. GARCH-M model was not found suitable for BSESN & CSE as the risk premium parameter of mean equation was found insignificant in all cases. It shows that there is no correlation between risk and return. Therefore, only GARCH and IGARCH model were built to in and out-sample forecasting for CSE and BSESN. The forecasting behaviour was evaluated in both ways in-sample; RMSE and MAE and out-sample; forecast root mean square errors (FRMSE).

The result shows that the BSESN is more persistent market as the sum of the coefficients of and of GARCH (1,1) model is greater than among all and KSE-100 is less persistent than other two. As we know that financial markets are very sensitive as they are affected to rumour, political upheavals, changes in government monetary and fiscal policies etc. Karachi Stock market is largest stock market of Pakistan during the study period due to unstable political situation it was less persistent that may be one of the reason.According to the Table 8, based on Ferecast root mean square error (FRMSE) we conclude that GARCH-M(1,1)-Log Variance for KSE-100, IGARCH(1,1) for CSE and for BSESN GARCH(1,1)/IGARCH(1,1) model is the most appropriate model for modelling the volatility .

Policy implication and Future research

This study is only in the context of selected stock markets of SAARC countries and these results cannot be generalized for all the member countries. Basically, Economic health is associated with stable stock markets. Therefore, good governance, better monetary policy, low unemployment rate and increasing FDI will improve the investment in the Pakistan stock market. Increase investment is associated with stability of stock market. This study is focuses on univarite symmetric volatility models to compare the performance of Srilanka, India, and Pakistan stock markets. In future, asymmetric univarite GARCH models and multivariate frame work approaches may be applied to investigate the leverage effect and co-movements of price change in these markets.

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